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Comparing Aircraft Agility Using Mahalanobis Distances

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Introduction

IN a conversation among pilots, one might hear a statement to the effect that, considering agility, this airplane is "head and shoulders" above that one. The purpose of this Note is to show that such notions of distance can be quantified by use of the Mahalanobis generalized distance measure. Even though the presentation that follows is couched in terms of aircraft agility, the method is generally applicable.

Suppose a set of p agile maneuvers $X_1, X_2, \ldots, X_p$ is executed and measured N_1 times on an aircraft of one type, and N_2 times on an aircraft of another type, for a total of $N = N_1 + N_2$ p-variate measurements, all correlated and each measured in possibly different units. It is assumed that the errors of measurement are multivariate normal with equal covariance matrices. We want to calculate from these measurements a set of dimensionless distances D between the two aircraft for each of the p maneuvers, and for a linear compound of all the maneuvers.

In 1936 Mahalanobis[1] devised a generalized distance measure D defined by

$$D^2 = \boldsymbol{d}'S^{-1}\boldsymbol{d} \tag{1}$$

where $\boldsymbol{d}$ is a vector of differences between the p means, $\boldsymbol{d}'$ is the transpose of $\boldsymbol{d}$, and S^{-1} is the inverse of the pooled variance-covariance matrix S. Interest in and use of this distance measure continues, especially in India, to the present time. We note that the Mahalanobis distance is measured in a nonorthogonal p-dimensional space.

In the case where $p = 1$, D can be calculated easily from the equivalent relation

$$D^2 = [N(N - 2)R^2]/[N_1N_2(1 - R^2)] \tag{2}$$

where R is the sample correlation coefficient. For example, suppose the first aircraft performs one maneuver three times, and records the values 1, 2, and 3. The other aircraft performs the same maneuver three times and records the values 4, 5, and 6. We introduce a dummy variable y consisting of zeros for the first aircraft, and ones for the other aircraft, and display the data in two columns, as shown in Table 1.

We used a hand-held calculator, the Texas Instruments TI-35X, and entered the six data pairs in Table 1. With a single key-stroke, the calculator displayed the correlation coefficient $R = 0.87831$. Using Eq. (2) we got $D^2 = 9$. When there are p maneuvers, this simple procedure can be used to get a distance between the two aircraft for each one of the p maneuvers. The statistical significance of each of these distances can be checked by comparing the calculated R with a statistical table of R. Some of these values of R may be statistically nonsignificant, in which case one might be tempted to delete the data from that maneuver. However, this could be a mistake, because any distance measured, however small or statistically insignificant, may be highly suggestive, and therefore useful. For further discussions of the dangers of discarding correlated data, see Kramer[2] and Kendall.[3]

The defining relationship Eq. (1) can also be used to calculate D^2. From the data above, the mean difference $d = 5 - 2 = 3$. We calculated the pooled variance to be 1. Substituting in Eq. (1) we got $D^2 = 3 \times 1 \times 3 = 9$ as before. Either Eq. (1) or Eq. (2) may be used if we wish to calculate the distance between the two aircraft based on a linear compound (called a linear discriminant function) of the p measurements, and a computer already programmed for this is recommended. A superb choice is a stepwise buildup regression program (without an intercept estimate), which enters one variable at a time into the regression (namely that variable which most reduces the regression error) and prints a multiple

Table 1 Data for one maneuver

Dummy variable y	Maneuver X_1
0	1
0	2
0	3
1	4
1	5
1	6

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Table 2 Data for two maneuvers

Dummy variable y	Maneuver X_1	X_2
0	0.58579	2
0	2	2
0	3.41421	2
1	5	4.58579
1	5	6
1	5	7.41421

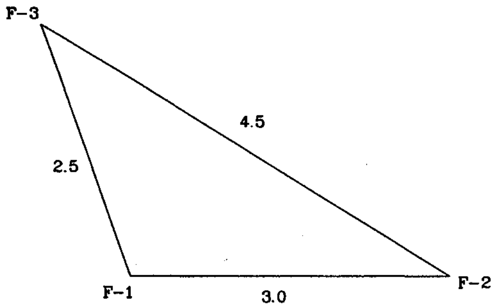

Fig. 1 Agility distance between three aircraft.

correlation coefficient R at each step, which can be used in Eq. (2). Importantly, this program also calculates a statistical test for the significance of the set of remaining variables not yet entered into the regression. It should be noted that the vector of regression coefficients calculated at each step is equal to the vector $d'S^{-1}$ which appears in Eq. (1) (see Ref. 2), so that Eq. (1) could be used as well to compute the Mahalanobis distance at each step.

Furthermore, let D_p^2 and D_q^2 denote the squared Mahalanobis distances between two aircraft based on p maneuvers and any subset of q maneuvers, respectively. It can be shown[4] that, in general, $D_p^2 \geq D_q^2$. Thus, if we have calculated a Mahalanobis distance between two aircraft for each maneuver, we would select from these distances the largest one as the best indicator of distance between these aircraft based on a single maneuver. (It would be a mistake to average these distances.) Thus, the different distances between aircraft based on one maneuver tell us which single maneuver best, or least, reveals an agility difference. Again, however, the least revealing maneuver should not be discarded, not only for the reasons noted above, but also because that maneuver may be more revealing for other aircraft in future tests.

For the case $p = 2$, Kramer[2] gives an example completely worked out for real data from an industrial experiment. Pedantically, in order to facilitate computation, we have constructed hypothetical data as shown in Table 2.

Using Eq. (1), we easily get $D^2(X_1 \ \& \ X_2) = (3 \ \ 4)\ (I)\ (3 \ \ 4)' = 25$, where I is the identity matrix, and $D^2(X_1 \ \& \ X_2)$ is the squared Mahalanobis distance based on both X_1 and X_2. Similarly, $D^2(X_1) = 9$ and $D^2(X_2) = 16$, where both of these last results also can be obtained easily using Eq. (2).

Suppose there are three aircraft, the F-1, F-2, and F-3, with a calculated distance of 3.0 between the F-1 and the F-2, 2.5 between the F-1 and the F-3, and 4.5 between the F-2 and the F-3. These results can be shown on a triangle as in Fig. 1.

If there are four aircraft and a calculated distance between each pair, the results can be similarly shown on a tetrahedron, which makes an interesting and revealing visual display.

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Effect of a Fuselage on Delta Wing Vortex Breakdown

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Nomenclature

b = wing span
c_r = root chord of exposed wing panel
D = fuselage diameter
K_b = upwash factor for vortex breakdown
K_W = slender-body-theory Beskin upwash factor
Re_{c_r} = Reynolds number, based on root chord
x = axial distance measured aft of apex of exposed wing panel
x_b = axial distance to mean breakdown position measured aft of apex of exposed wing panel
α_∞ = angle of attack of freestream flow relative to wing root chord
$\alpha_{b,w}$ = angle of attack of wing-alone configuration for breakdown at x_b/c_r
$\alpha_{b,\mathrm{wb}}$ = angle of attack of wing-body configuration for breakdown at x_b/c_r
α_{eq} = equivalent angle of attack
α_0 = angle-of-attack offset
Λ_w = leading-edge sweep angle

Introduction

THE importance of leading-edge vortex flows to slender aircraft has been well-documented in recent years in many reports.[1–3] As a result, the aerodynamic characteristics of leading-edge vortex flows have been extensively studied both experimentally and computationally. Much of this research has been in the area of vortex breakdown, the sudden and dramatic change that results in the turbulent dissipation of the vortex. Various investigators[4–8] have reported the many factors influencing the vortex breakdown on delta wings. Some of those factors include aspect ratio, camber, cropping, flaps, surface roughness, and trailing-edge geometry. Most of the previous work has reported on wing-alone (without fuselage)

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